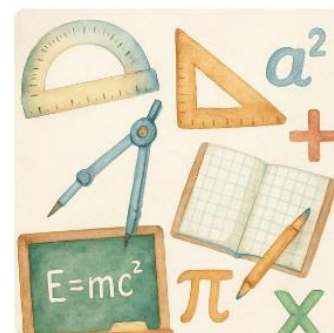

CH X – SYSTEMS OF EQUATIONS, AN INTRODUCTION

Here's an example of a **system of two equations in two variables**:

$$\begin{aligned}x + y &= 10 \\ 2x - 3y &= 5\end{aligned}$$

The *two equations* are easy to see, as are the *two variables*. The term *system* refers to the fact that these two equations are tied together — our final solution must be a pair of numbers, one for x and one for y , satisfying both equations.



❑ WHAT DOES IT MEAN TO BE A SOLUTION OF A SYSTEM OF EQUATIONS?

In the system above, the values $x = 8$ and $y = 2$ will satisfy the first equation [$8 + 2 = 10$ ✓], but will not satisfy the second equation [$2(8) - 3(2) = 16 - 6 = 10$, not 5]. Therefore, $x = 8$ and $y = 2$ is not a solution of the *system* of equations.

Sticking with the same system of equations above, the values $x = -8$ and $y = -7$ will satisfy the second equation [$2(-8) - 3(-7) = -16 + 21 = 5$ ✓] but will not satisfy the first equation [$-8 + (-7) = -15$, not 10]. Therefore, $x = -8$ and $y = -7$ is another pair of numbers which do not constitute a solution of the *system* of equations, either.

But $x = 7$ and $y = 3$ is a solution of the system of equations. Here's why:

$$7 + 3 = 10 \quad \checkmark \quad \text{and} \quad 2(7) - 3(3) = 14 - 9 = 5 \quad \checkmark$$

□ WHERE CAN SUCH A SYSTEM OF EQUATIONS ARISE?

How does such a system of equations come about? How about a problem like the following:

Find two numbers whose sum is 17 and whose difference is 1.

If you wanted to solve this problem using Algebra, you would proceed something along these lines: First give names to the two unknown numbers; maybe call them g and h .

Then the *sum* part of the problem yields the equation

$$g + h = 17$$

The *difference* part of the problem would imply that

$$g - h = 1 \quad \text{[We will assume that } g > h.\text{]}$$

We now have a **system** of two equations in two variables:
$$\begin{array}{l} g + h = 17 \\ g - h = 1 \end{array}$$

Among the variety of methods available to solve such a system, pay attention to the method required by your instructor.

Homework

1. Consider the system of equations:
$$\begin{array}{l} a + b = 9 \\ a - b = 7 \end{array}$$
 - a. Show that $a = 5$ and $b = 4$ is a solution of the first equation, but is not a solution of the system.
 - b. Show that $a = 20$ and $b = 13$ is a solution of the second equation, but is not a solution of the system.
 - c. Show that $a = 8$ and $b = 1$ is a solution of the system.

2. Try to solve the system $\begin{array}{l} u + w = 12 \\ u - w = 0 \end{array}$ by guessing.

Now try the system $\begin{array}{l} 3x - 17y = 200 \\ -5x - 12y = 29 \end{array}$ by guessing (just kidding!).

3. Give an example of a system of three equations in three variables.
4. Translate the following word problem to a system of two equations in two variables: *The sum of two numbers is 25 and their product is 150.*
5. Translate the following word problem to a system of two equations in two variables: *The difference of two numbers is 25 and their quotient is 6.*

Solutions

1. a. $5 + 4 = 9$ ✓ But $5 - 4 = 1 \neq 7$.
 b. $20 - 13 = 7$ ✓ But $20 + 13 = 33 \neq 9$.
 c. $8 + 1 = 9$ ✓ And $8 - 1 = 7$ ✓
2. Both variables = 6, since $6 + 6 = 12$, and $6 - 6 = 0$.
3. Something like

$$\begin{array}{rcl} 3x - 2y + z & = & 9 \\ 5x + y + 10z & = & 44 \\ -4x - 5y - z & = & -2 \end{array}$$

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4. $x + y = 25$
 $xy = 150$

[Of course, you can use any two variables you'd like.]

5. $a - b = 25$
 $\frac{a}{b} = 6$

[We're assuming that $a > b$ and that $b \neq 0$.]



**“To educate a
man in mind and
not in morals is to
educate a menace
to society.”**

Theodore Roosevelt
(1858–1919)